$$\vec{\nabla}\,\vec{A} = -\mu_0\varepsilon_0 \frac{\partial V}{\partial t}$$

$$-\vec{\nabla}^2 \vec{A} + \mu_0\varepsilon_0 \frac{\partial^2 A}{\partial t^2} = \mu_0 \vec{J}$$

$$-\vec{\nabla}^2 V + \mu_0\varepsilon_0 \frac{\partial V}{\partial t} = \frac{\rho}{\varepsilon_0}$$

no source
no current) in vacuum

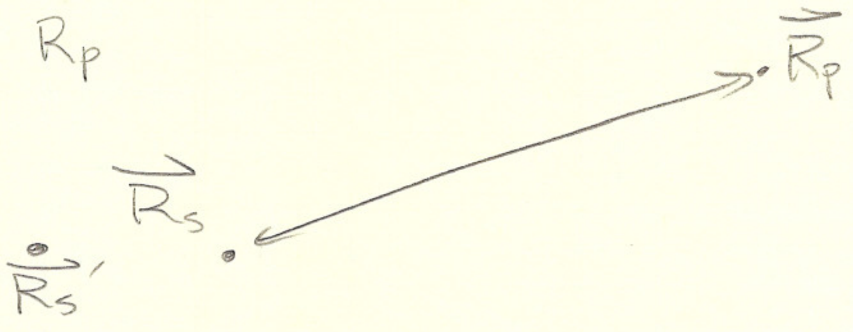
$$\vec{\nabla}^2 \vec{A} = \mu_0\varepsilon_0 \frac{\partial^2 \vec{A}}{\partial t^2}$$

$$\vec{\nabla}^2 V = \mu_0\varepsilon_0 \frac{\partial^2 V}{\partial t^2}$$

$$\mu = \frac{1}{\sqrt{\mu_0\varepsilon_0}} = c$$

finite

At R_p



It takes a finite time to go from

$\vec{R_s}$ to $\vec{R_p}$. It takes more time for the feild created by $\vec{R_s}'$ to reach $\vec{R_p}$ then for $\vec{R_s}$. Therefore we must consider the different points at different times. (Retarded Potential)

For a plane wave $\sim e^{jKR}$

$$= e^{jk|\vec{R_p} - \vec{R_s}|}$$

$$= e^{jkc\Delta t}$$

In electrostatics.

$$V(\vec{R_p}) = \frac{1}{4\pi\varepsilon_o}\int_V dV \rho(\vec{R_s}) \frac{1}{|\vec{R_p} - \vec{R_s}|}$$

....

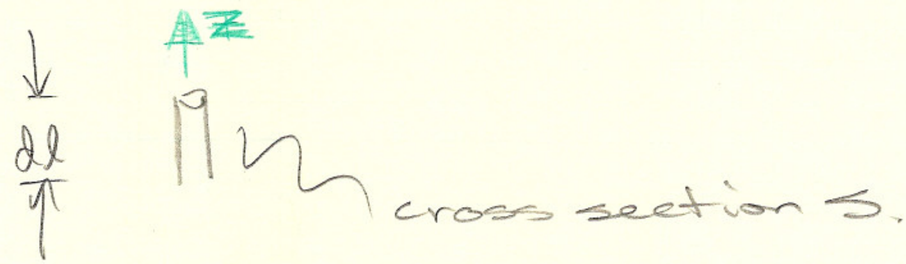
$$V_s(\vec{R_p}) = \frac{1}{4\pi\varepsilon_o}\int_{Vol} dV \rho_s(\vec{R_s}) \frac{e^{-jk|\vec{R_p} - \vec{R_s}|}}{|\vec{R_p} - \vec{R_s}|}$$

$$\vec{A_s}(\vec{R_p}) = \frac{-\mu_o}{4\pi\varepsilon_o}\int dV\, I_s(\vec{R_s}) \frac{e^{-jk|R_p - R_s|}}{|\vec{R_p} - \vec{R_s}|}$$

$$\vec{H}_s = \frac{1}{\mu_0} \vec{\nabla} \times \vec{A}_s(\vec{R}_p)$$

$$= \frac{1}{\eta} \hat{k} \times \vec{E}_s$$

Hertzian Dipole



$$\vec{J}_s(\vec{R}_s) = \frac{I_0}{S} \hat{z}$$

$$\vec{R}_s = 0$$

$$\vec{A}_s(\vec{R}_p) = \frac{\mu_0}{4\pi} \int_{-\frac{dl}{2}}^{\frac{dl}{2}} dl_s I_0 \hat{z} \frac{e^{-jk|\vec{R}_p - \vec{R}_s|}}{|\vec{R}_p - \vec{R}_s|}$$

$$\boxed{\int dV = s \int d\ell}$$

$$|\vec{R_p} - \vec{R_s}| = \sqrt{(\vec{R_p} - \vec{R_s}) \cdot (\vec{R_p} - \vec{R_s})}$$

$$= \sqrt{R_p^2 - 2R_pR_s\underbrace{\cos\theta}_{\text{angle b/t } R_p \text{ \& } R_s} + R_s^2}$$

$$= R_p\left[1 - \frac{2R_s\cos\theta}{R_p} + \frac{R_s^2}{R_p^2}\right]^{1/2}$$

Binomial theorem

$$\cong R_p\left[1 - \frac{R_s}{R_p}\cos\theta + \cancelto{0}{\frac{1}{2}\frac{R_s^2}{R_p^2}}\right]$$

$R_s \lll R_p$

$$\therefore\ |\vec{R_p} - \vec{R_s}| = R_p - R_s\cos\theta$$

$$\vec{A}(\vec{R_p}) = \frac{\mu_0}{4\pi}\int_{-\frac{d\ell}{2}}^{\frac{d\ell}{2}} d\ell_s\, I_0\,\hat{z}\,\frac{e^{-jk|\vec{R_p} - \vec{R_s}|}}{|\vec{R_p} - \vec{R_s}|}$$

$$\cong \frac{\mu_0}{4\pi}\int_{-\frac{dl}{2}}^{\frac{dl}{2}} dl_s I_0 \hat{z} \frac{e^{-jkR_p}}{R_p}$$

$$\cong \hat{z}\frac{\mu_0}{4\pi} I_0 \frac{e^{-jkR_p}}{R_p}(dl)$$

$$\hat{z} = \hat{r}\cos\theta_p - \hat{\theta}\sin\theta_p$$

$$\vec{A}_s(\vec{R_p}) = A_{rs}\hat{r} + A_{\theta s}\hat{\theta}$$

$$A_{rs} = \frac{\mu_0}{4\pi} dl\, I_0 \frac{e^{-jkR_p}}{R_p}(\cos\theta_p)$$

$$A_{\theta s} = \frac{\mu_0}{4\pi} dl\, I_0 \frac{e^{-jkR_p}}{R_p}(-\sin\theta_p)$$

$$\mu_0\vec{H}_s(\vec{r_p}) = \vec{\nabla}\times\vec{A}_s(\vec{R_p})$$

$$= \frac{1}{R_p^2\sin^2\theta_p}\begin{bmatrix} \hat{r} & R_p\hat{\theta} & \hat{\phi}R_p\sin\theta_p \\ \partial/\partial R_p & \partial/\partial\theta_p & \partial/\partial\phi_p \\ A_{rs} & R_pA_{\theta s} & R_p\sin\theta_p A_{\phi s} \end{bmatrix}$$

$$= \hat{\phi}\frac{\mu_0}{4\pi} dl\, I_0\, k^2 \frac{e^{-jkR_p}}{R_p}\sin\theta_p\left[\frac{j}{kR_p} + \frac{1}{(kR_p)^2}\right]$$

$$\vec{E}_s(\vec{R}_p) = \frac{1}{j\omega\varepsilon_o} \vec{\nabla} \times \vec{H}_s(\vec{R}_p)$$

Taking the curl again...

$$= \frac{1}{j\omega\varepsilon_o} \left\{ \hat{r} \left[\frac{2 I_o dl}{4\pi} k^2 \eta_o e^{-jkR_o} \cos\theta_p \left[\frac{1}{(kR_p)^2} - \frac{j}{(kR_p)^3} \right] \right] \right.$$

$$\left. + \hat{\theta} \left[\frac{I_o dl}{4\pi} k^2 \eta_o e^{-jkR_p} \sin\theta_p \left[\frac{j}{kR_p} + \frac{1}{(kR_p)^2} - \frac{j}{(kR_p)^3} \right] \right] \right\}$$

$$\vec{H}_s = \phi \frac{1}{4\pi} dl\, I_o k^2 \frac{e^{-jkR_p}}{R_p} \frac{j\sin\theta_p}{kR_p}.$$

$$\vec{E}_s = \hat{\theta} \frac{j I_o dl}{4\pi} k \eta_o \frac{e^{-jkR_p}}{R_p} \sin\theta_p$$

$$\therefore \quad \vec{H}_s = \hat{\phi} \frac{E_{os}}{\eta_o}$$